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USE OF INTEGRAL BOUNDARY-LAYER THEORY FOR SOLVING CONJUGATE PROBLEMS OF HEAT TRANSFER IN CHANNELS OF HIGH-POWER PLANTS

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The solution of the conjugate problem of convective-conductive heat transfer in the channels of power plants is presented. The problem of convective heat transfer in the gas phase is solved by the integral theory of heat transfer. A one-dimensional problem of conductive heat transfer in the material of the wall is solved by the finite-difference method. Relative laws of heat and mass transfer and friction are obtained by numerical integration with respect to the boundary-layer thickness. The effect of the material and geometry of the wall on the level of problem "conjugation" is studied.

Introduction. Mass ideality of the heat protection of high-power plants (HPPs) can be reached using most developed methods of calculation of heat and mass transfer. Modern HPPs are characterized by a wide range of operating times (0.1–300 sec), the presence of high-intensity heat fluxes (the temperatures of fuel combustion vary from 1500 to 4000 K), considerable elongation of the channel (L/d = 0.5–100), and nonstationarity of processes in a working body (start-up, control) and in the material of the uncooled wall. In modeling the thermal state of an HPP, a conjugate formulation of the problem becomes necessary due to the calculation of temperature fields in the wall material (especially when materials susceptible to thermal shock are used) and evaluation of heat losses of the working body (the most urgent in elongated systems of gas distribution).

Formulation of the Problem. In spite of the long-standing attempts by different authors to solve the issue of the measure of the effect of conjugation in equations of convective-conductive heat transfer, this problem, being nontrivial, still requires consideration in each specific case. Therefore, we study the relation between conjugation and the parameters characterizing the heat transfer between a gas and a solid body for the cases occurring in the practice of HPP design.

According to [1], the necessity of solving problems in a conjugate formulation can be evaluated by the Brun number introduced as a criterion of conjugation

$$\mathrm{Br}_x = \frac{\lambda_f}{\lambda_s}\frac{b}{x}\mathrm{Pr}^{0.4}\mathrm{Re}^{0.8} .$$

It is assumed that for $\mathrm{Br} < \mathrm{Br}_{\mathrm{crit}}$ the problem can be solved in a nonconjugate formulation with sufficient accuracy. A. V. Luikov [1] suggested to determine $\mathrm{Br}_{\mathrm{crit}}$ on the basis of comparison with experiment or with an accurate analytical solution. Since it is impossible to obtain an analytical solution for the general case of a turbulent gas flow past a wall, then consideration of the Brun number as a quantitative criterion is of no practical value, but it may be useful as a qualitative criterion.

In the analysis of quantities entering into the Brun number we can distinguish the component, $\frac{\lambda_f}{x}\mathrm{Pr}^m\mathrm{Re}^n$, which is responsible for flow in a boundary layer. It becomes clear from this component that in order to analyze the effect of the material and the structural features of the wall on problem conjugation, one

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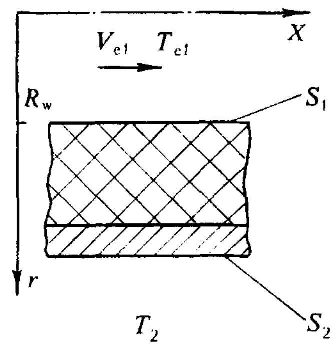

Fig. 1. Scheme of a flow past the HPP wall.

should ensure identical conditions in the boundary layer and at the conjugation point, i.e., study flow of the same gas under identical initial conditions, with the same coordinate X, but in combination with different walls. As such model flow we consider subsonic flow of combustion products in a channel of a model solid-fuel gas generator operating at constant pressure.

Mathematical Model. A mathematical model of the conjugate problem of nonstationary convective-conductive heat transfer for a gas–solid system (see Fig. 1) is written as:

Gas phase. Taking into consideration that the flow is forced and the transfer of substances along X greatly exceeds the transfer along r, we can assume that the flow is subdivided into two regions: the region of inviscid flow which (in a one-dimensional approximation) is described by the Euler equations

$$\rho_e V_e \frac{\partial V_e}{\partial x} + \rho_e \frac{\partial V_e}{\partial t} = -\frac{\partial p}{\partial x}, \quad \rho_e V_e \frac{\partial h_e^*}{\partial x} + \rho_e \frac{\partial h_e^*}{\partial t} = \frac{\partial p}{\partial t},$$

and the boundary-layer region described by the Prandtl equations

$$\rho \frac{\partial V_x}{\partial t} + \rho V_x \frac{\partial V_x}{\partial x} + \rho V_r \frac{\partial V_x}{\partial r} = -\frac{\partial p}{\partial x} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\mu \frac{\partial V_x}{\partial r}\right),$$

$$\rho \frac{\partial h^*}{\partial t} + \rho V_r \frac{\partial h^*}{\partial r} + \rho V_x \frac{\partial h^*}{\partial r} = \frac{\partial p}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\mu}{\mathrm{Pr}} \frac{\partial h^*}{\partial r}\right) + \frac{1}{r}\frac{\partial}{\partial r}\left[r \frac{\mu(\mathrm{Pr}-1)}{\mathrm{Pr}} \frac{\partial}{\partial r}\left(\frac{V_x^2}{2}\right)\right].$$

For the entire flow region, the continuity equation must hold:

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r V_r) + \frac{\partial}{\partial x}(\rho V_x) = 0.$$

The system is closed by the equation of state of an ideal gas

$$p = \rho R T.$$

Since already in the beginning of the duct of the considered HPPs at x = 5 mm the number $\mathrm{Re}_x = \rho_e V_e x/\mu$ is 2500, i.e., it exceeds a critical number $\mathrm{Re}_{cr} \approx 2200$, the laminar portion can be neglected and flow along the duct of the HPP can be assumed to be turbulent. Then the system of Prandtl equations can be considered as a system of averaged equations.

Solid body. The process of energy transfer in a solid body is described by the equation of heat conduction

$$c\rho \frac{\partial T}{\partial \tau} = \mathrm{div}\,(\lambda\, \mathrm{grad}\, T) + q_v.$$

To close this system, one should set boundary-value conditions.

Initial conditions ($\tau = 0$):

a gas phase

$$V(x, r) = V_{\text{in}}(x, r), \quad h^*(x, r) = h^*_{\text{in}}(x, r), \quad p(x) = p_{\text{in}}(x);$$

a solid body

$$T(x, r) = T_{\text{in}}(x, r).$$

Boundary conditions:

$$x = x_0: \quad V(r) = V_0(r, \tau), \quad h^*(r) = h^*_0(r, \tau), \quad p = p_0(\tau);$$

$$r = R_{\text{w}} - \delta: \quad V_x|_{r=R_{\text{w}}-\delta} = V_{\text{e}};$$

$$r = R_{\text{w}} - \delta_{\text{h}}: \quad h^*|_{r=R_{\text{w}}-\delta_{\text{h}}} = h^*_{\text{e}};$$

boundary S_1

$$V_x = 0, \quad (\rho V)_r = j_{\text{w}}, \quad h^* = h_{\text{w1}}, \quad -\lambda \frac{\partial T_{\text{s}}}{\partial r} = q_{\text{conv}} + q_{\text{rad1}},$$

boundary S_2

$$-\lambda \frac{\partial T_{\text{s}}}{\partial r} = q_{\text{rad2}} + q_{\text{n.conv2}},$$

where h_{w1} is the gas enthalpy calculated at the wall temperature T_{w1}, and j_{w} is the mass flow rate of the products of pyrolysis obtained by solution of the equation of heat conduction.

With the entire possible spectrum of perturbation factors that are present in the actual structures (nonisothermicity, injection, longitudinal gradient of pressure, gradient of stagnation enthalpy, dynamic and thermal nonstationarity, surface roughness, longitudinal curvature), the use of laborious methods of convective heat transfer (k–ε, etc.) seems inexpedient for solving conjugate problems. Integral methods are traditionally assumed to be most universal, simple in use, and possessing the highest, among other methods of convective heat transfer, index (accuracy/labor consumption) [2]. Therefore, in the present work we use a method based on the solution of boundary layer equations in the form of integral relations in combination with integration of the system of equations with respect to the boundary-layer thickness for obtaining relative laws of heat and mass transfer and friction.

The system of equations for the initial section has the following form:

the momentum equation

$$\frac{H'D}{V_{\text{e}}} \frac{\partial \operatorname{Re}^{**}}{\partial t} + \frac{\partial \operatorname{Re}^{**}}{\partial \bar{x}} + \operatorname{Re}^{**} \frac{(H+1)}{V_{\text{e}}} \frac{\partial V_{\text{e}}}{\partial \bar{x}} + \operatorname{Re}^{**} \frac{(H - H')\, D}{V_{\text{e}}^2} \frac{\partial V_{\text{e}}}{\partial t} +$$

$$+ \frac{\operatorname{Re}^{**}}{R_0} \frac{\partial R_0}{\partial \bar{x}} = \left(b_{\text{w}} + \frac{C_{\text{f}}}{2} \right) \operatorname{Re}_D; \qquad (1)$$

the energy equation

$$\frac{H_h' D}{V_e} \frac{\partial \mathrm{Re}_h^{**}}{\partial t} + \frac{\partial \mathrm{Re}_h^{**}}{\partial \bar{x}} + \frac{H_h' \mathrm{Re}_h^{**} D}{V_e (h_w^* - h_w)} \frac{\partial h_w^*}{\partial t} - \frac{H_h' \mathrm{Re}_h^{**} D}{V_e (h_w^* - h_w)} \frac{\partial h_w}{\partial t} +$$

$$+ \frac{\mathrm{Re}_h^{**}}{h_w^* - h_w} \frac{\partial h_w^*}{\partial x} - \frac{\mathrm{Re}_h^{**}}{h_w^* - h_w} \frac{\partial h_w}{\partial x} + \frac{(H - H') \mathrm{Re}^{**} D}{V_e h_w^* - h_w} \frac{\partial h_w^*}{\partial t} +$$

$$+ \frac{H \mathrm{Re}^{**}}{h_w^* - h_w} \frac{\partial h_w^*}{\partial x} - \frac{H_h' \mathrm{Re}_h^{**} D}{V_e^2} \frac{\partial V_e}{\partial t} + \frac{\mathrm{Re}_h^{**}}{R_0} \frac{\partial R_0}{\partial x} = (b_w + \mathrm{St}) \, \mathrm{Re}_D ; \tag{2}$$

the equation of flow rate

$$\frac{D (H' - H)}{V_e} \frac{\partial \mathrm{Re}^{**}}{\partial t} - H \frac{\partial \mathrm{Re}^{**}}{\partial \bar{x}} + \frac{D (H - H') \mathrm{Re}^{**}}{V_e^2} \frac{\partial V_e}{\partial t} +$$

$$+ \frac{R_0 \mathrm{Re}_D}{2 V_e D} \frac{\partial V_e}{\partial \bar{x}} + \frac{R_0 \mathrm{Re}_D}{2 \rho_e V_e} \frac{\partial \rho_e}{\partial t} + \frac{R_0 \mathrm{Re}_D}{2 \rho_e D} \frac{\partial \rho_e}{\partial \bar{x}} -$$

$$- \frac{H \mathrm{Re}^{**}}{R_0} \frac{\partial R_0}{\partial \bar{x}} + \frac{\mathrm{Re}_D}{D} \frac{\partial R_0}{\partial \bar{x}} = b_w \, \mathrm{Re}_D ; \tag{3}$$

the equation of motion for the inviscid core of the flow

$$\rho_e \frac{\partial V_e}{\partial t} + \rho_e V_e \frac{\partial V_e}{\partial x} = - \frac{\partial p}{\partial x} , \tag{4}$$

the energy equation for the inviscid core of the flow

$$\rho_e \frac{\partial h_e^*}{\partial t} + \rho_e V_e \frac{\partial h_e^*}{\partial x} = \frac{\partial p}{\partial t} . \tag{5}$$

Relations (1)-(5) in combination with the equation of state of an ideal gas compose a closed system of equations relative to six unknowns: V_e, ρ_e, p, h_e^*, Re^{**}, and Re_h^{**}.

Expressions (1)-(3) hold their form for the transition region and the region after the joining of the boundary layers, although two additional equations are required to close the system. For this purpose, the method described in [3] is used.

The system of equations for determination of the relative laws of heat and mass transfer and friction is written separately. This system allows one, on each step along the longitudinal coordinate X, to calculate the values of the relative laws reflecting the difference between the actual boundary layer (under the effect of the entire set of perturbation factors) and the model boundary layer. Thus, we obtain a system of partial differential equations of the form

$$\frac{\partial f}{\partial \tau} = \frac{\partial f}{\partial x} + g(f) ,$$

where $f(x, \tau)$ is the unknown function (V_e, ρ_e, h_e^{**}, Re^{**}, Re_h^{**}).

Two ways of solving this system are possible: 1) solution of the system of partial differential equations by the finite-difference method; 2) use of a quasistationary method of "bands" where a system of ordinary nonlinear differential equations of the form $\partial f / \partial x = g(f)$ is solved for specific instants of time τ_1, τ_2, ... , and

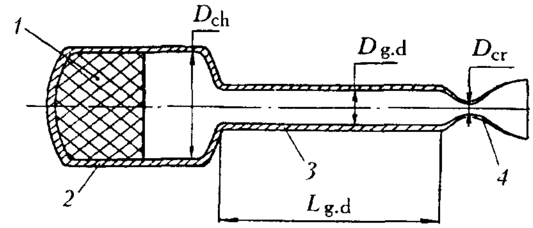

Fig. 2. Schematic diagram of a model HPP: 1) source of a working body; 2) chamber; 3) gas duct; 4) Laval nozzle.

TABLE 1. Versions of Walls for Model Studies

Number of layer	Version 1		Version 2		Version 3	
	material	h, mm	material	h, mm	material	h, mm
1	Glass fiber plastic material	5	High-temperature steel	3	Carbon-carbon composite material	5
2	–	–	Glass fiber plastic material	2	–	–

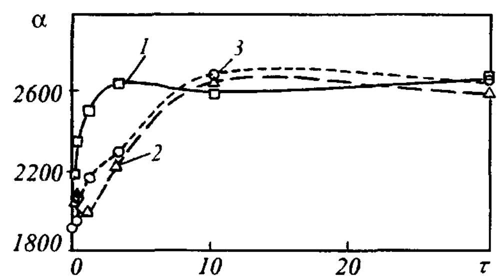

Fig. 3. Influence of wall material on the coefficient of heat transfer (from the solution of the conjugate problem): 1-3) versions. α, W/(m^2·K); τ, sec.

τ_k with the second way being more simple and more efficient, since the system of equations can be solved by the Runge–Kutta method, the Adams method, and other methods which possess good convergence and stability. In this case, the integral boundary-layer theory allows one to take into account the effect of nonstationarity directly on the relative laws of heat and mass transfer and friction.

A temperature field in the material of the structure wall can be obtained on the basis of a one-dimensional nonstationary equation of heat conduction, for the solution of which we use a modification of the finite-difference method based on the use of the reference-volume method.

Convective and conductive parts of the problem are conjugated within the framework of the iteration procedure with respect to the temperatures and heat fluxes at the phase interface.

Discussion of Results. A schematic diagram of a model HPP is given in Fig. 2. The initial temperature of the wall was assumed to be equal to 283 K, the temperature of the working body was 1300 K, the working pressure in the chamber was 40 atm, and the time of reaching the mode was 0.1 sec.

We conducted three numerical experiments, in which walls corresponding to limiting versions of the range of Brun numbers were studied and all materials and geometries of the walls were selected proceding from the actual designs (see Table 1).

Figures 3 and 4 present results of calculation of a conjugate problem of heat transfer for the three versions mentioned above. The results are given in the form of curves of the relative temperatures, with the coordinate $y < 0$ corresponding to the wall, and $y > 0$ to the boundary layer.

The thickness of the boundary layer in all the versions was equal to about 5 mm, which made it possible to construct the temperature profiles with the absolute coordinate y for an instant of 0.3 sec that charac-

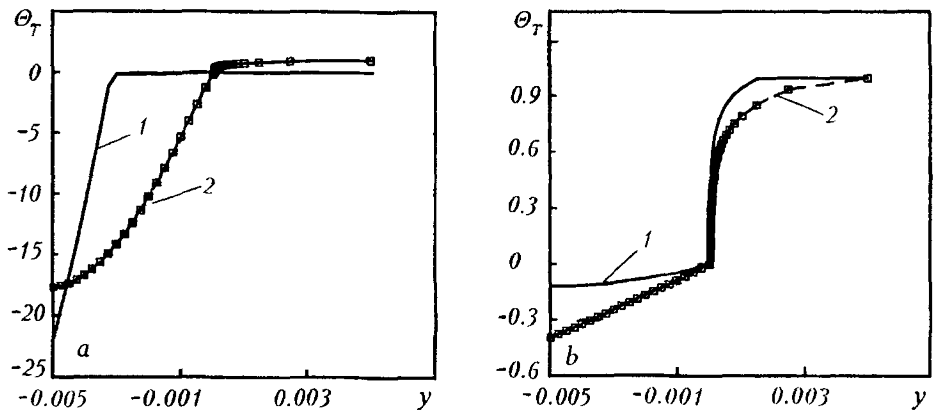

Fig. 4. Distribution of the temperature fields in a wall ($y < 0$) and in a boundary layer ($y > 0$): a) 1) version 1 (30 sec, Br = 982); 2) version 2 (30 sec, Br = 17); b) version 3 [1) 0.3 sec, Br = 7; 2) 30 sec, Br = 15]. y, m.

terizes the period of start of the gas generator with a "cold" structure, and for an instant of 30 sec that corresponds to about the middle of the operating cycle of the engine when the structure is already heated.

The large difference in values of the Brun number, which is equal to 982 for a wall made of pure glass-fiber-plastic material and 7 for a wall made of highly heat-conducting carbon-carbon composite material (CCCM), should be noted. A combined metal–glass-fiber-plastic wall occupies an intermediate position.

The highest difference of temperatures over the thickness of the wall is observed for version 1 with a solid glass-fiber-plastic wall; the difference is $\Theta_T > 20$. Moreover, Br = 982, which, according to [1], indicates the highest conjugation among all the cases considered. However, if we analyze the curves of the coefficient of heat transfer α (Fig. 3), we see that the coefficient of heat transfer for glass fiber plastic reaches values close to asymptotic ones very quickly. This indicates that for this case, especially when $\tau > 1$ sec, it is reasonable to use a nonconjugate formulation and to solve the problems for a gas and a solid body separately.

An opposite picture is observed for version 3 (a solid wall made of CCCM). The Brun numbers do not exceed 15, and the relative temperature difference over the wall thickness is smaller than 0.2, which, according to [1], indicates a comparatively low degree of problem conjugation. However, it becomes clear from a consideration of the curves that the coefficient of heat transfer approaches an asymptotic value comparatively slowly. Moreover, in this version a considerable discharge of energy by radiation from the exterior wall is present, which is in no way allowed for by the number Br. Consequently, this wall cannot be considered as isothermal and the problem given should be solved in a conjugate formulation with iteration joining with respect to the heat fluxes.

The case of a combined wall occupies an intermediate position with respect to Br numbers. At the initial instant, heat is expended on heating a metal layer with rather high thermal conductivity, which leads to a comparatively small temperature difference over the wall, smaller than 0.2. However, by the 30th second, due to the redistribution of heat over the thickness of glass fiber plastic the temperature difference reaches values much higher than unity. An analysis of the curves of α shows that they approach asymptotic values rather slowly, thus indicating the necessity of solving the problem in a conjugate formulation. Moreover, in contrast to the case with pure glass fiber plastic, in this version the conjugation of the problem will change greatly as the wall is heated.

Conclusions. The study conducted allows one to draw the conclusion that the Brun number does not reflect all subtleties of heat transfer processes in HPP and in a number of cases (a wall made of CCM, a glass-fiber-plastic wall), an analysis of conjugation on the basis of the Brun number can lead to opposite conclusions. Nevertheless, for simple cases, the Brun number Br can be used for a preliminary estimation as a qualitative criterion of conjugation.

NOTATION

δ, boundary-layer thickness; ΘT, relative temperature, where $\Theta = (T - T_w)/(T_e^* - T_w)$ for the solid body and $\Theta = (T^* - T_w)/(T_e^* - T_w)$ for the boundary layer; τ, shearing stress; $C_f = 2\tau_w/(\rho_e V_e^2)$, coefficient of friction; $h^* = h + V_x^2/2$, stagnation enthalpy; $h_{re}^* = h + r(V_x^2/2)$, enthalpy of recovery; $\mathrm{Re}^{**} = (\rho_e V_e \delta^{**})/\mu$, characteristic Reynolds number; Re_D, Reynolds number constructed from the channel diameter; $\mathrm{St} = q_w/(\rho_e V_e[h_{re}^* - h_w])$, Stanton number; $\Delta h_1 = h_{re}^* - h_w$, difference of enthalpies; q_v, source component in the equation of heat conduction; D, characteristic dimension, channel diameter. Subscripts: s, solid body; f, fluid; e, boundary of the boundary layer; 0, model boundary layer; w, wall; h, thermal boundary layer; 1, parameter at the boundary S_1; 2, parameter at the boundary S_2; in, initial; ch, chamber; g.d, gas duct; cr, critical; conv, convection; n.conv, natural convection; rad, radiative; crit, criterial.

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